

Three Key Findings

1. Exponent's Statistical Analysis of the Difference in Average Pressure Drops is Wrong Because it Ignores Timing.
2. Exponent Improperly Draws Conclusions Based on the Variability in Halftime Pressure Measurements Despite Conceding that the Variability is Statistically Insignificant.
3. If the Logo Gauge was Used to Measure the Patriots Balls Before the Game, Then 8 of 11 were *Above* Exponent's Expected Outcome.



Finding 1:

Exponent's "Most Significant" Statistical
Analysis Ignores Timing

Raw Data

Table 1. The pressure readings taken of footballs during halftime of the AFC Championship Game on January 18, 2015 (pressures are listed in psig).

Team	Ball	Tested by Clete Blakeman	Tested by Dyrol Prioleau
Patriots	1	11.50	11.80
	2	10.85	11.20
	3	11.15	11.50
	4	10.70	11.00
	5	11.10	11.45
	6	11.60	11.95
	7	11.85	12.30
	8	11.10	11.55
	9	10.95	11.35
	10	10.50	10.90
	11	10.90	11.35
Colts	1	12.70	12.35
	2	12.75	12.30
	3	12.50	12.95
	4	12.55	12.15

Source: Exponent Report, Table 1

Raw Data and PSI Averages by Team and Official

Table 2. Pressure measurements of the footballs as recorded on Game Day.

Team	Ball	Tested by Clete Blakeman	Tested by Dyrol Prioleau
Patriots	1	11.50	11.80
	2	10.85	11.20
	3	11.15	11.50
	4	10.70	11.00
	5	11.10	11.45
	6	11.60	11.95
	7	11.85	12.30
	8	11.10	11.55
	9	10.95	11.35
	10	10.50	10.90
	11	10.90	11.35
<i>Patriots Average</i>		<i>11.11</i>	<i>11.49</i>
Colts	1	12.70	12.35
	2	12.75	12.30
	3	12.50	12.95
	4	12.55	12.15
	<i>Colts Average</i>		<i>12.63</i>

Source: Exponent Report, Table 2

Exponent Scenario 3

Table 5. Data taken during halftime with Colts Measurement 3 adjusted and attributed to the two different gauges.

Team	Ball	Non-Logo Gauge	Logo-Gauge
Patriots	1	11.50	11.80
	2	10.85	11.20
	3	11.15	11.50
	4	10.70	11.00
	5	11.10	11.45
	6	11.60	11.95
	7	11.85	12.30
	8	11.10	11.55
	9	10.95	11.35
	10	10.50	10.90
	11	10.90	11.35
<i>Patriots Average</i>		<i>11.11</i>	<i>11.49</i>
Colts	1	12.35	12.70
	2	12.30	12.75
	3	12.50	12.95
	4	12.15	12.55
	<i>Colts Average</i>		<i>12.33</i>

“Note that for the remainder of this report, when the Game Day data are discussed, it should be taken to mean that the data from Scenario 3 are the data being referenced.” (Exponent Report, p. 12)

Source: Exponent Report, Table 5

Difference in Average Pressure Drops: Exponent Scenario 3

“What is most significant about the halftime measurements is that ***the magnitude of the reduction in average pressure was greater for the Patriots footballs when compared to that of the Colts footballs.*** The question then becomes: what factor(s) could explain this difference?”

Table 6. Averages from both Logo and Non-Logo Gauges, assuming the gauge switch between measurements for each team and the reversal of the Colts third measurement, with pressure drops calculated.

Gauge	Patriots Average	Average Pressure Drop (based on initial reading of 12.50 psig)	Colts Average	Average Pressure Drop (based on initial reading of 13.00 psig)	Difference in Pressure Drops between Colts and Patriots (psig)
Non-Logo Gauge	11.11	1.39	12.33	0.67	0.72
Logo Gauge	11.49	1.01	12.74	0.26	0.75

Sources: Exponent Report, Table 6 and p. IX (emphasis added)

“Difference in Differences” Statistical Approach Used by Exponent

- Calculated differences between pre-game and halftime PSIs for Patriots and Colts balls.
- Using Colts balls as controls, inquired whether the greater drop of PSI in Patriots balls was statistically significant.
- Adopted statistical significance threshold of 0.05 to evaluate “difference in differences.”
- Presented conclusions.

Source: Exponent Report, pp. 4-12

Exponent's Conclusions From Difference in Differences Statistical Analysis

Table 8. The p-values calculated using Exponent's statistical model for each of the four Game Day halftime data scenarios.

Scenario	p-value
1	0.004
2	0.004
3	0.004
4	0.017

“As shown in Table 8, the p-value for all of the above scenarios is less than 0.05. This indicates that regardless of which of the above four sets of assumptions are made about the gauges used to generate the Game Day data, the difference between the average pressure drop of the Patriots and Colts footballs is statistically significant. ***In other words, in all cases studied, the additional pressure drop exhibited by the Patriots footballs is unlikely to have occurred by chance.***”

Sources: Exponent Report, Table 8 and p. 11 (emphasis added)

Timing of Colts Ball Measurement Matters

- “According to basic thermodynamics, it is completely expected that the temperature and pressure inside a football drop when it is brought from a warmer environment into a colder environment and rise when brought back into a warmer environment. ***It is important to note, however, that these variations in temperature and pressure are time-dependent (in the time ranges at issue in the present investigation).***” (Exponent Report, pp. X, 64 (emphasis added))
- “[A] ***key factor in explaining the difference in measurements between the Patriots and Colts footballs is timing***; that is, the change in pressure with time as the footballs were brought from a colder environment (the field) to a warmer environment (the Officials Locker Room) at halftime.” (Exponent Report, pp. XII, 66 (emphasis added))

Timing of Colts Ball Measurement Matters

- “[T]he change in pressure inside the balls when brought back into the locker room from the field is strongly dependent on time. ***Specifically, the pressure in a football measured immediately after coming into the locker room will be significantly lower as compared to the pressure measured in the same football once it has sat (and warmed up) in the locker room for several minutes.***” (Exponent Report, p. 43, emphasis added)
- “That there is a strong time dependence of the pressure inside the footballs upon return to the simulated locker room is of significant impact in the present investigation.” (Exponent Report, p. 43)

Exponent's Figure 22: Time Simulation Results

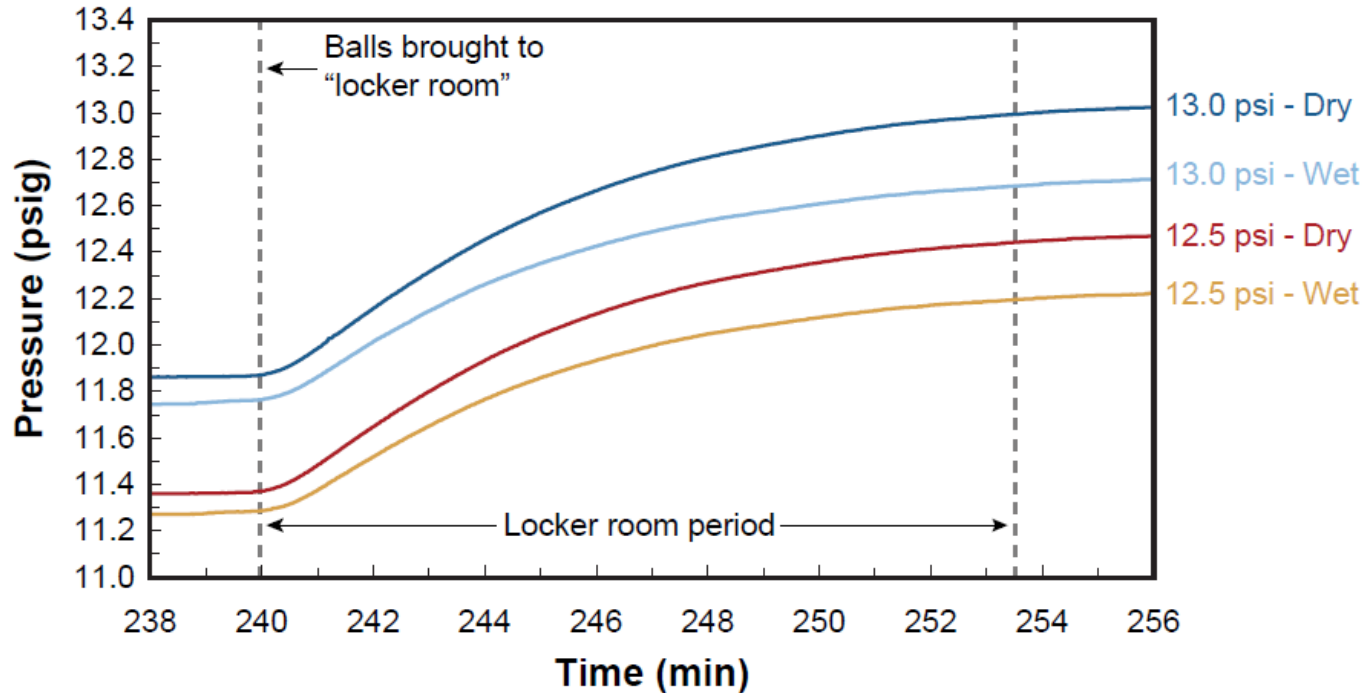
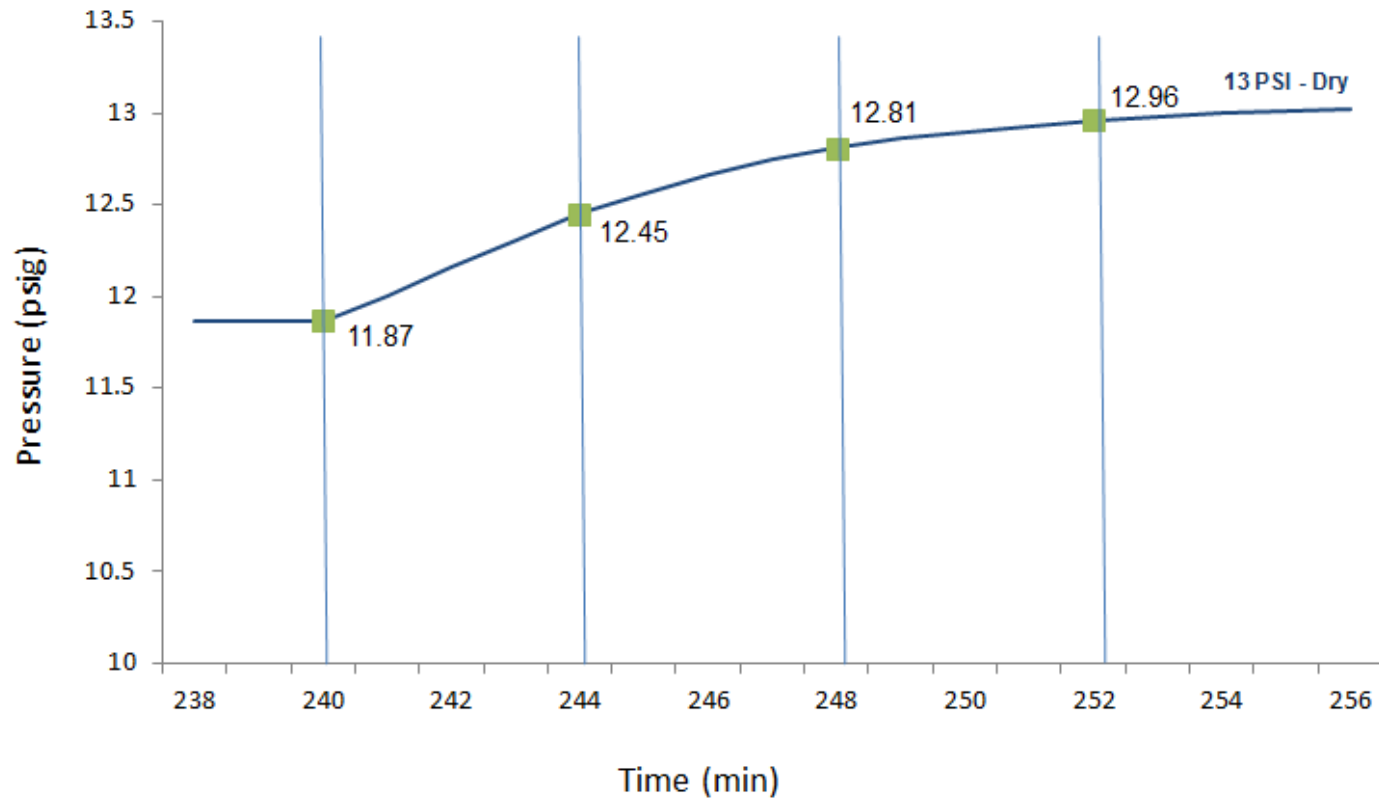


Figure 22. The "Locker Room" portion of the transient curves (first shown in Figure 21).

Source: Exponent Report, Figure 22

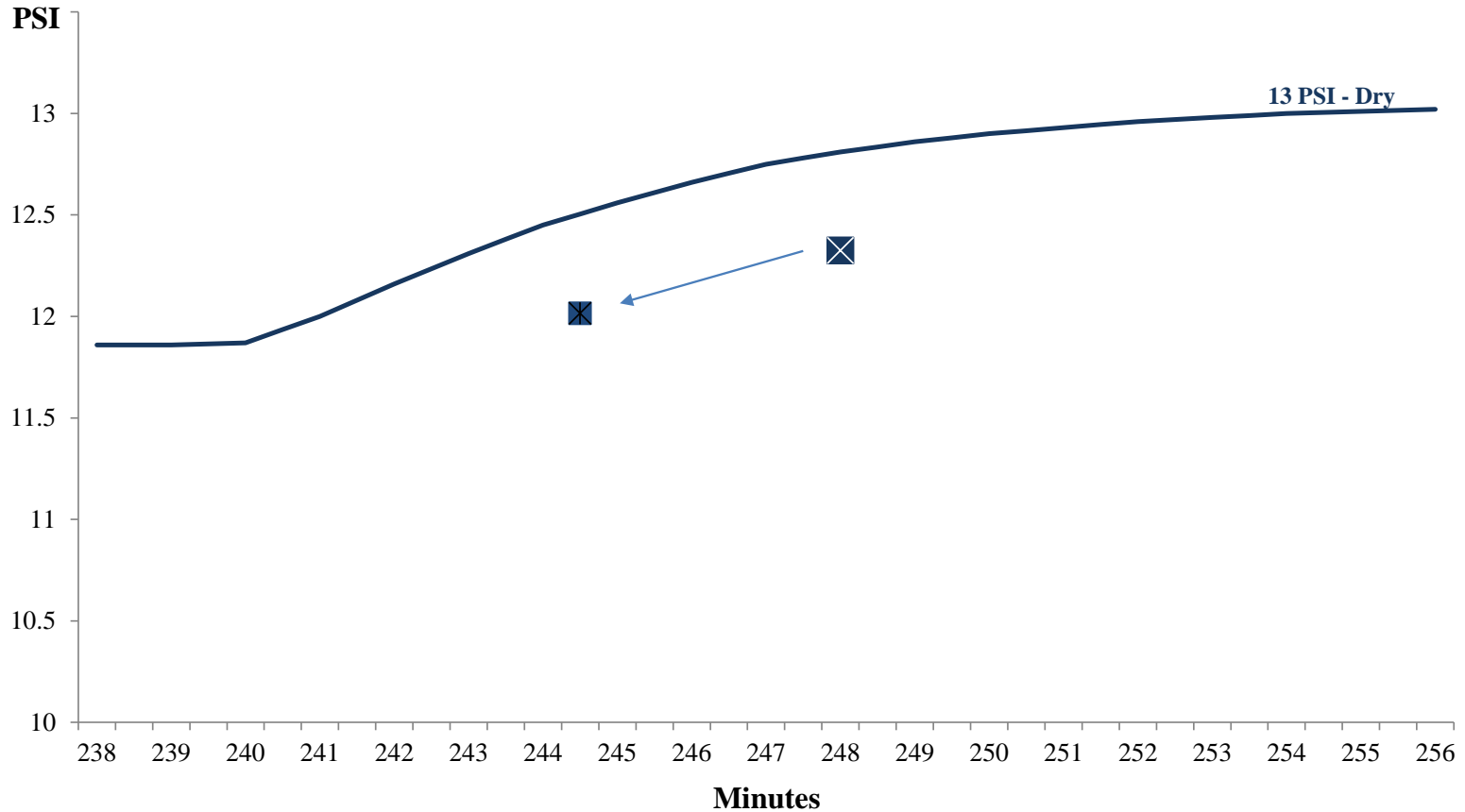
Exponent's Figure 22 Showing A Colts Ball Measured At Different Points In Time



Make Apples-to-Apples Comparisons Controlling for Time

- **Case 1:** Measurement of Colts balls immediately after measurement of Patriots balls (Exponent's assumption).
- **Case 2:** Measurement of Colts balls at the end of halftime (Exponent's alternative).
- **Case 3:** Case 2, now accounting for drier Colts balls if tested at the end of halftime.

Case 1 – Exponent’s Assumption: Colts Balls Measured Immediately after Patriots Balls



Source: Exponent Report, Figure 22

Case 1 – Exponent’s Assumption: Colts Balls Measured Immediately after Patriots Balls

	Ignoring Timing (Exponent)	Accounting for Timing
Difference in differences	0.72 – 0.75	0.41 – 0.44
p-value	0.004	0.067

Case 2: Colts Balls Measured After Reinflation of Patriots Balls

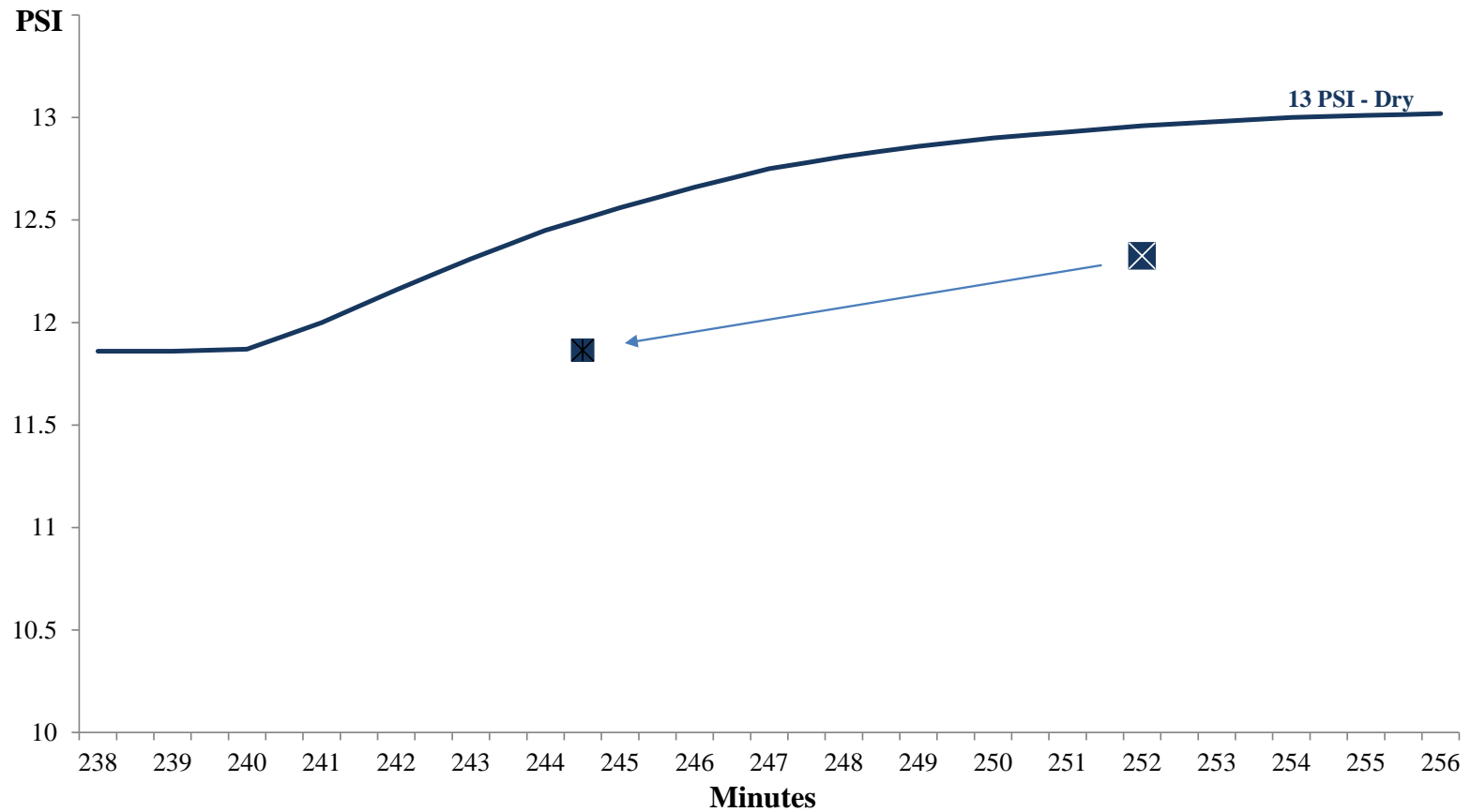
“According to information provided by Paul, Weiss, during the halftime period, three events pertaining to the footballs are known to have occurred:

1. The air pressure measurements of 11 Patriots footballs were taken and recorded.
2. The air pressure measurements of four Colts footballs were taken and recorded.
3. The reinflation and regauging of 11 Patriots footballs to a level within the 12.5–13.5 psig range was performed.

According to information provided by Paul, Weiss, it is clear that of the three events listed above, the measuring of the Patriots balls occurred first. Although ***there remains some uncertainty about the exact order and timing of the other two events***, it appears likely the reinflation and regauging occurred last.”

Source: Exponent Report, p. 2 (emphasis added)

Case 2: Colts Balls Measured After Reinflation of Patriots Balls

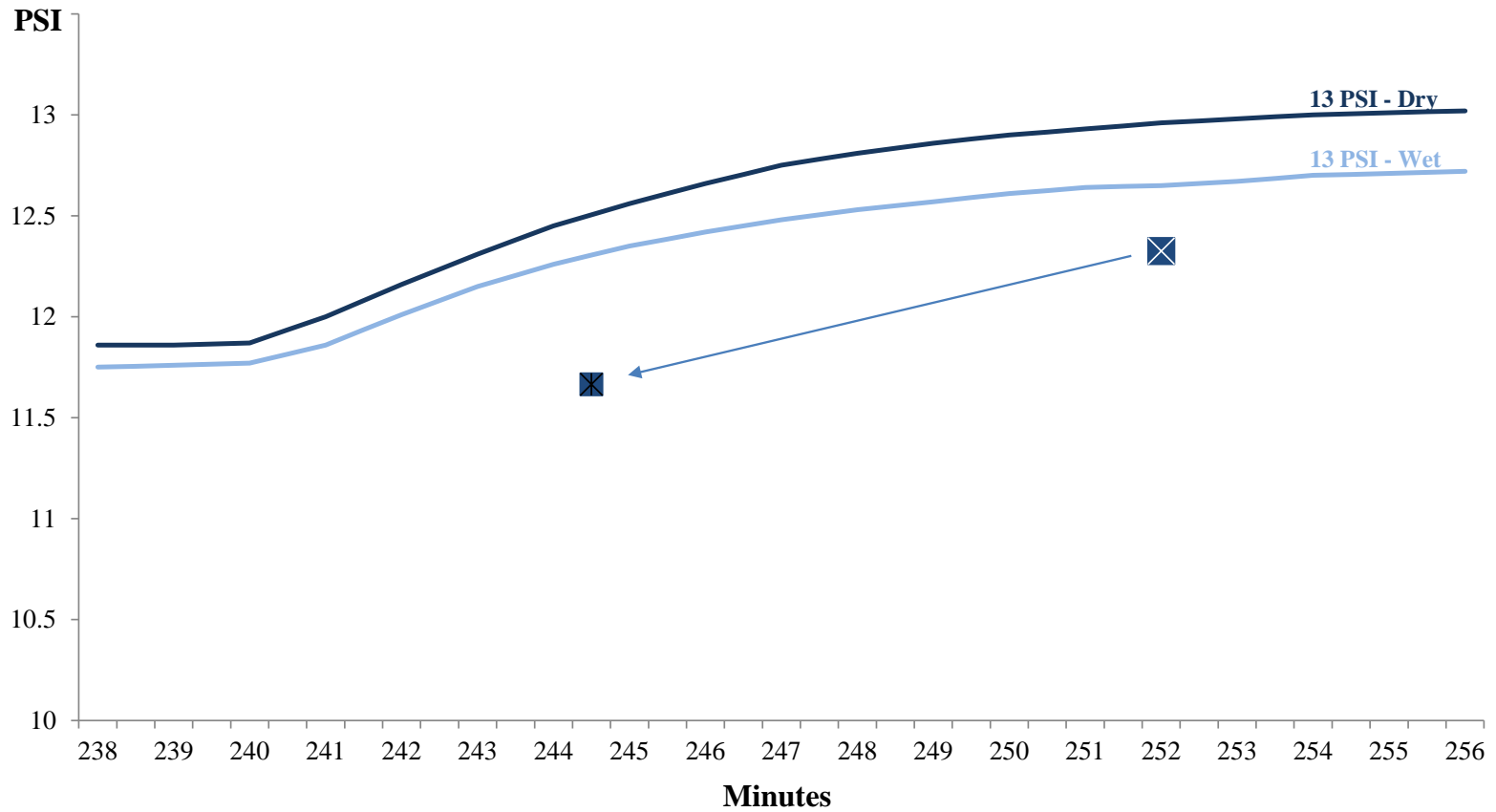


Source: Exponent Report, Figure 22

Case 2: Colts Balls Measured After Reinflation of Patriots Balls

	Ignoring Timing (Exponent)	Accounting for Timing
Difference in differences	0.72 – 0.75	0.26 – 0.29
p-value	0.004	0.219

Case 3: Colts Balls Measured After Reinflation of Patriots Balls Dry to Wet



Source: Exponent Report, Figure 22

Case 3: Colts Balls Measured After Reinflation of Patriots Balls Dry to Wet

	Ignoring Timing (Exponent)	Accounting for Timing
Difference in differences	0.72 – 0.75	0.06 – 0.09
p-value	0.004	0.734



Finding 2:

Exponent Improperly Draws Conclusions
from its Statistical Variability Analysis

Conclusion about Variability in Halftime Pressure Measurements

“In addition to noting the difference in average pressure drops between the Colts and Patriots footballs when measured at halftime, we observed that there appears to be a difference in the variability of the measurements recorded for each team. Although ***we found the difference between the variability in halftime pressure measurements of the Patriots and the Colts footballs not to be statistically significant***, we can draw certain conclusions on variability when these data are reconsidered in the context of our experimental results.”

“Specifically, the fluctuations in the halftime pressures of Patriots footballs exceed in magnitude the fluctuations that can be attributed to the combined effects of the various physical, usage, and environmental factors we examined. ***Therefore, subject to discovery of an as yet unidentified and unexamined factor, it is our view that the most plausible explanation for the variability in the Patriots measurements recorded at halftime is that the 11 Patriots footballs measured by the officials at halftime did not all start the game at or near the same pressure.***”

Sources: Exponent Report, pp. XIV and 68 (emphases added)

Exponent's Figure 22: Time Simulation Results

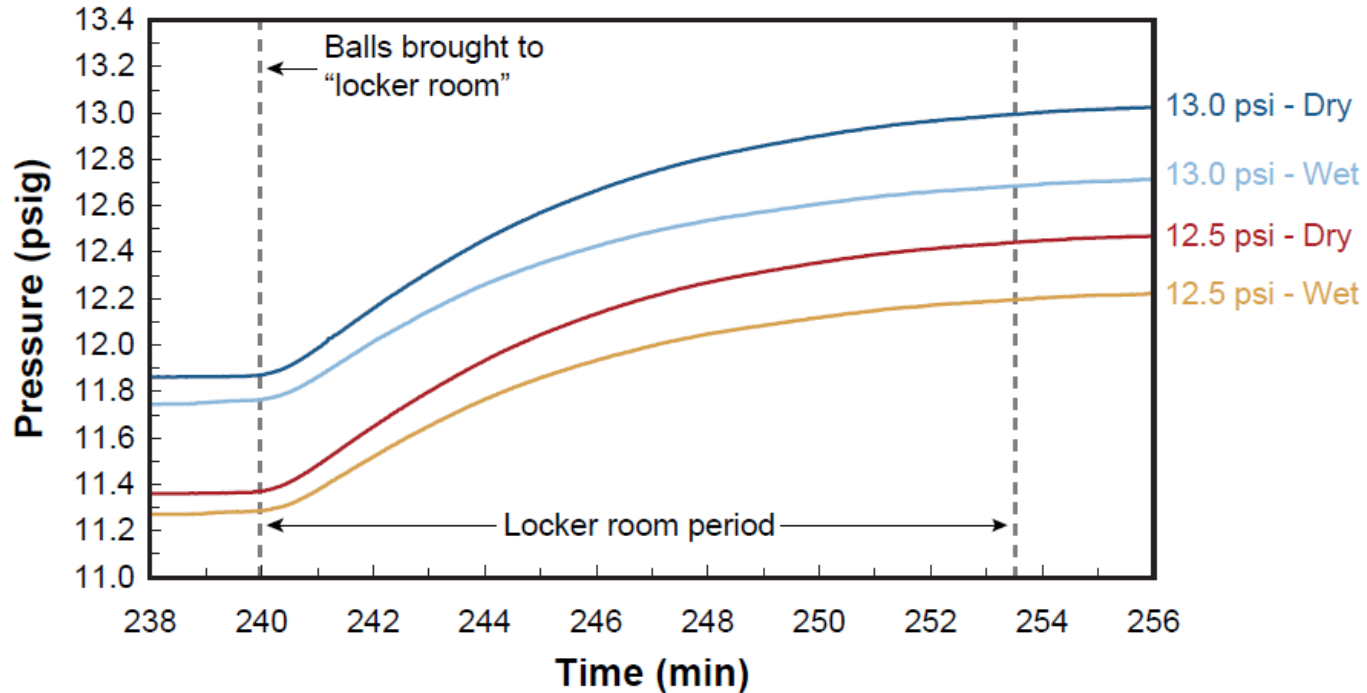


Figure 22. The "Locker Room" portion of the transient curves (first shown in Figure 21).

Source: Exponent Report, Figure 22



Finding 3:

If the Logo Gauge was Used Pre-Game,
then 8 of 11 Patriots Balls were *Above*
Exponent's Expected Outcome

Exponent's Ideal Gas Law Calculation

“For example, using the most likely pressure and temperature values for the Patriots game balls on the day of the AFC Championship Game (i.e., **a starting pressure of 12.5 psig**, a starting temperature of between 67 and 71°F and a final temperature of 48°F prior to the balls being taken back into the Officials Locker Room), these equations predict that the Patriots balls **should have measured between 11.52 and 11.32 psig** at the end of the first half, just before they were brought back into the Officials Locker Room.”

Source: Exponent Report, p. 40 (emphases added)

Exponent Misapplies Patriots Balls Measurements In Ideal Gas Law Formula

- ***“Once the game day measurements are converted into their corresponding Master Gauge pressures*** (in order to provide for a direct comparison with the results predicted by the calculations). . .” (Exponent Report, p. 40, emphasis added)
- Exponent concludes that 8 out of 11 balls are below the lower bound of the range. (Exponent Report, p. 40)
- Yet Exponent does not make Master Gauge conversion to adjust initial PSI reading. (Exponent Report, Table 10)

Pre-Game Measurements Using the Logo Gauge

- “Although [Referee] *Anderson’s best recollection is that he used the Logo Gauge*, he said that it is certainly possible that he used the Non-Logo Gauge.” (Wells Report p. 52, emphasis added)
- “We have been told by Paul, Weiss that there remains some uncertainty as to which of the two gauges was used prior to the game.” (Exponent Report, p. IX)
- “[T]he Logo Gauge generally overestimated the Master Gauge by ~0.3-0.4 psig. . .” (Exponent Report, p. 20)

Correcting Initial PSI of Patriots Balls Using Master Gauge Conversion Formula

- Applying Exponent's Master Gauge conversion formula, the initial PSI for the Patriots Balls would actually have been 12.17 – not 12.5 – if the Logo Gauge was used pre-game. (Exponent, p. 28)

Corrected Results

Inputting an initial PSI of 12.17 to the Ideal Gas Law formula, 8 of 11 Patriots balls are **above** the lower bound of the expected range.

	Initial PSI Not Master-Gauge Adjusted	Initial PSI Master-Gauge Adjusted
	Lower Bound of Expected Ideal Gas Law PSI	
	11.32	11.01
Number of Patriots Balls		
Above	3	8
Below	8	3

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